A retro-review
Merks, J W, 2005

Geostatistical Ore Reserve Estimation
David, M, 1977
Elsevier Scientific Publishing Company

In the *Preface* to this first textbook on geostatistics, Professor D A Krige, the pioneering plotter of distance-weighted averages at the Witwatersrand gold reef complex in South Africa, confessed, *"Having been associated intimately with* [its] *birth and early development"*, credits Matheron and David with its development and establishment, and reminisces about, *"some stormy receptions in English-speaking mining countries around the world."* David himself cautions, *"This is not a book for professional statisticians"*, presciently predicts, *"statisticians will find many unqualified statements",* and feels compelled to apologize to, *"our statistical readers"*.

Most troubling to statistically astute readers are the facts that the variance of a ***single*** distance-weighted average vanished, and that it is replaced with the variance of a ***set*** of distance-weighted averages. Ironically, this sort of pseudo variance of a ***set*** of distance-weighted averages is as a meaningless a measure for variability, precision and risk as its pseudo covariance is for spatial dependence. These fundamental errors, and scores of others, are scattered throughout a text richly embellished with geostat speak, krige-inspired eponyms, atypical symbols and confusing neologisms such as deconvolution, non-independent, nugget effect, sill value and smoothing.

In *Chapter 1* under *1.4.1 Definition of independence,* the author alludes to spatial dependence between ordered data but ignores the difference between independently measured and functional dependence. Only independently measured values, unlike functionally dependent *(calculated)* values, give degrees of freedom. In mathematical statistics, each functionally dependent value (the arithmetic mean or some weighted average) has its own variance. In geostatistics, however, the distance-weighted average metamorphosed into a kriged estimates and its variance vanished without a trace.

In *Chapter 3 What is an ore reserve calculation?*, the author shows why geostatistics cannot possibly provide unbiased confidence limits for metal contents and grades of ore reserves as a measure for risk. Under *"Real"* and *"Kriged"* both Table 3.I and Table 3.II list independently measured data and kriged estimates. The author declares, *"We see that the kriging procedure in this case is much better; the spread, or variance, of the distribution is much smaller."* Mathematical statisticians accept that *n* independently measured values give *n–1* degrees of freedom, and that a functionally dependent value (distance-weighted average-*cum*-kriged estimate) has its own variance. Calculating pseudo variances of **sets** of functionally dependent values is not just abuse of statistics but a genuine scientific fraud.

Chapter 10 The Practice of Kriging, makes it perfectly clear that the distance-weighted average at a selected position is a functionally dependent value of a set of independently measured values at different positions in a sample space. *Figure 203* on page 286 explains that all 16 points within sample space *B* are ***"estimated"*** from the same nine holes. In mathematical statistics, each *"point"* is a functionally dependent value simply because it is ***calculated*** from the same nine holes.

The author does not show how to apply Fisher's F-test to verify whether or not his *in situ* ordered set of nine holes exhibits a significant degree of spatial dependence within its sample space. He mentions double- and half-variograms but does not explain the sampling variogram in which the variance terms of the ordered set are plotted against the variance of the randomized set and the lower limits of its asymmetric 99% and 95% confidence ranges to show where orderliness in a sample space dissipates into randomness.

On the same page the author pontificates, *"Writing all the necessary covariances for that system of equations is a good test to find out whether one really understands geostatistics!"* A good test to find out whether one really understands mathematical statistics is to count the degrees of freedom for the randomly distributed and ordered sets. If these nine holes were equidistant, then the randomly distributed *(randomized)* set would have $df_r=9–1=8$ degrees of freedom whereas the first variance term of the ordered set would have $df_o=2\cdot(9–1)=16$. The author appears unaware that degrees of freedom are positive integers for independently measured values with equal weights, and positive irrationals for independently measured values with variable weights.

In *2.1.1 The standard error of the mean*, the author extols *"the famous central limit theorem"* and admits it *"has been responsible for the largest number of mistakes on the account of statistics."* One such mistake is his failure to explain that the variances of all weighted averages converge on the central limit theorem as variable and equal weights converge. Somewhat surprisingly, the *"famous"* central limit theorem failed to make the index of this very first textbook on geostatistics.

Degrees of freedom materialize accidentally in a footnote in *Table 1.IV* but, unlike *aureola* (sic), *bull's eye shot* and *chaotic component*, degrees of freedom, too, failed to make the index. In *Example* on page 35, the author uses *t0.05;10=2.22* rather than *t0.05;9=2.262* to compute 95% confidence limits for the central value of a set of ten test results for iron, and misses a unique opportunity to link geostatistics not only to the concept of degrees of freedom but also to its roots in mathematical statistics.

Table 9.II in *Chapter 9 Optimization of the Grade Estimation: Kriging*, gives a set of paired test results for gold determined in bulk samples and in coincidental core samples. The author uses Sichel's t-estimator to develop a *"correction factor"* to predict bulk grades from core grades. Given that the difference between the mean bulk and core grades is statistically identical to zero, either set can be used to predict the other without

running a significant risk that the difference between the means of bulk and core grades becomes a bias. This is a classic example of abuse of statistics.

In *Chapter 12*, the author confirms, *"There is an infinite set of simulated values"*, and ponders how to, *"make that infinite set smaller and get the model closer to reality"*. He does not tackle the daunting task of selecting the least biased subset of an infinite set of conditionally simulated data and smoothing its pseudo kriging variance to perfection. In a rare instant of lucidity, the author confesses, *"The criticism to this model is obvious. The simulation is not reality. There is only one answer: The proof of the pudding is…!"* Were Bre-X's phantom gold resource and Hecla's shrinking Grouse Creek reserve some of David's proverbial pudding proofs?

In *Chapter 13*, the author recalls Gy's lifetime preoccupation with *"the variance of sampling errors"* but in *13.1.2 Bias Generation*, he proffers the bizarre claim that high variances generate bias. Perhaps not surprisingly because a*ccuracy* and *precision* are synonymous in his own textbook; a practice traceable to Gy's often quoted but highly impractical work. Various ISO standards do give proper definitions of *bias*, *accuracy* and *precision*.

In his *List of Notations*, the author boasts, *"It has been known for a long time that geostatisticians seem to have that capability of changing notations twice or more in the same page and still understand each other."* Geostatisticians also have the remarkable capability of reviewing mathematical statistics without understanding it. David was one of CIM reviewers who rejected Fisher's F-test for spatial dependence.

David's textbook proves beyond reasonable doubt that geostatistics is an invalid variant of mathematical statistics because it violates the fundamental requirement of functional independence and ignores the concept of degrees of freedom. It is an irrefutable fact that one-to-one correspondence between central values (arithmetic mean and weighted averages) and variances is *in sine qua non* in mathematical statistics. In geostatistics, however, one-to-one correspondence between distance-weighted averages and variances is null and void.

The variance of the distance-weighted average is replaced with the pseudo variance of a set of distance-weighted averages. All this gibberish seems to have made sense to Matheron because he conferred the ubiquitous krige-eponym on the pioneering plotter of distance-weighted averages who somehow lost the infinite set of variances of distance-weighted averages somewhere in South Africa. Regrettably, its rebirth as a kriged estimate did not cause its variance to vanish, nor did it make the distance-weighted average any less functionally dependent.